

Primes and subsets

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Summary

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In my Essay on number theory(first version: December 2018), I have found (I do not if it is known!!) an interesting relation between the set of subset of a finite set E , prime numbers and **Fermat** little theorem.

Corrected version.

If we think in term of sets: consider n as the cardinal of a finite set E , the number of subsets of E is known to be

$$|\mathcal{P}(E)| = 2^n.$$

For a given n , not all the subsets' cardinals : $n - 1, n - 2, \dots$ can be primes at the same time. So when we eliminate from $\mathcal{P}(E)$ the subsets:

- i) E
- ii) $\emptyset$
- iii) the singletons $\{x_i\}$

Or otherwise :

$$2^n = \binom{n}{0} + \binom{n}{1} + \binom{n}{2} + \dots + \binom{n}{n}$$

Then the number of subsets becomes

$$A = 2^n - n - 2$$

So we have this definition:

Definition:

An integer n is a prime if we cannot classify n objects into a finite number of sets E_i of the same cardinal > 1 such that we have for $k \neq l$:

$$E_k \cap E_l = \emptyset$$

There is a curious relation $\sim$ between n and this number A : **if n is prime** then it divides $2^n - n - 2$ namely

$$n | 2^n - n - 2$$

It is simply **Fermat's little theorem**:

$$2^n \equiv 2 \pmod{n}.$$

Since if p is a prime and $p | n$ then

$$p \leq \frac{n}{2}$$

we define two things:

- i) E_s : the number of subsets to check.
- ii) $|S|$: the cardinals of such subsets.

So the subsets' cardinals should rather verify

$$1 < |S| \leq \frac{n}{2}$$

It follows that the number of subsets we will check for the condition

$$\begin{cases} 1 < p, |S| \leq \frac{n}{2} \\ E_k \cap E_l = \emptyset \end{cases} \quad (*)$$

all verify the relation $\sim$ and are of four forms:

If **n is even** then E_s verifies the condition:

$$|E_s| < 2^{\frac{n}{2}} - \frac{n}{2} - 2$$

For example , when $n = 10$ there are $|E_s| < 25$ such sets.

If **n is odd** then E_s verifies one of the conditions:

$$|E_s| < 2^{\frac{n}{3}} - \frac{n}{3} - 2 , \quad |E_s| < 2^{\frac{n-1}{3}} - \frac{n-1}{3} - 2 ,$$

$$|E_s| < 2^{\frac{n-2}{3}} - \frac{n-2}{3} - 2$$

For example :

When $n = 15$, there are $|E_s| < 25$ subsets.

When $n = 37$, there are $|E_s| < 4082$ subsets.

When $n = 23$, there are $|E_s| < 119$ subsets.

The question is : for a given n , what is the exact number of the subsets to be checked for the condition (*) ?

$$|E_s| = ?$$